# A Vector Method for the Measurement of Positioning errors and Straightness Errors Over a Machine Work Volume

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## Abstract

To achieve higher position accuracy of a machine tool, it is important to measure the volumetric errors, including the linear positioning errors and straightness errors of all three axes and to compensate the volumetric errors provided that the machine tool is repeatable. With the latest generation of CNC controls it is now possible to achieve higher quality even on a lower cost machine. To do this it is important to measure the volumetric errors of the machine tool and to compensate these errors. Many of the controls available today have this capability but very few take advantage of this technology. Described here is a vector method developed by Optodyne. Using the Optodyne laser calibration system and the vector method, the volumetric errors, e.g. 3 linear positioning errors, 6 straightness errors and 3 squareness errors, can be measured. The measured volumetric errors can be used to compensate the machine errors and achieved higher volumetric accuracy. The time required to compensate the machine using the vector method is significantly less than that using conventional measurement procedures making it easier for manufactures to take advantage to the advancements in CNC control technology.

# I. Introduction

To achieve higher position accuracy of a machine tool, it is important to measure the volumetric errors, including the linear positioning errors and straightness errors of all three axes. Measuring all these errors and then compensating these errors can improve the machine accuracy, provided that the machine is repeatable (Ref. 1, 2 and 3). The key is how to measure these errors accurately and quickly. There are many methods to measure these errors (Ref. 4 and 5). However, all of these methods are very complex and time consuming.

Described here is a vector method. It can measure all these errors, using a simple and portable laser interferometer or a Laser Doppler Displacement Meter (LDDM<sup>TM</sup>) (Ref. 6), in 4 setups and within a few hours.

# II. Body Diagonal Displacement Measurement

To measure the displacement accuracy of a linear axis, a laser interferometer can be used. The laser beam is aligned to be parallel to the motion of the linear axis and the position errors are measured at each increment. Since the measurement direction is parallel to the direction of the movement, the measured displacement errors are not sensitive to the straightness errors, which are perpendicular to the displacement direction.

It is noted that for a quick check of volumetric accuracy, linear displacement measurement along 4 body diagonals is recommended by the B5.57 standard (Ref 7). This is because the body diagonal measurements are sensitive to all the errors such as the displacement errors, straightness errors, squareness errors and angular errors. Hence it is a good check of volumetric accuracy. However, if the measured errors are large, there is not enough data to identify the error sources.

# **III.** Vector Measurement

The basic concept of the vector method is that the laser beam direction (or the measurement direction) is not parallel to the motion of the linear axis. Hence, the measured displacement errors are sensitive to the errors both parallel and perpendicular to the direction of the linear axis. More precisely, the measured linear errors are the vector sum of errors, namely, the displacement errors (parallel to the linear axis), the vertical straightness errors (perpendicular to the linear axis), and horizontal straightness errors (perpendicular to the linear axis), and horizontal straightness errors (perpendicular to the linear axis), the vertical straightness error direction), projected to the direction of the laser beam. Furthermore, collect data with the laser beam pointing in 3 different diagonal directions; all 9-error components can be determined. Since the errors of each axis of motion are the vector sum of the 3 perpendicular error components, we call this measurement a "vector" method.

For conventional body diagonal measurement all 3 axes move simultaneously, the displacement is a straight line along the body diagonal; hence a laser interferometer can be used to do the measurement. However, for the vector measurement described here,

the displacements are along the x-axis, then along the y-axis and then along the z-axis. The trajectory of the target or the retroreflector is not parallel to the diagonal direction. The deviations from the body diagonal are proportional to the size of the increment X, Y, or Z. A conventional laser interferometer will be way out of alignment even with an increment of a few mm.

To tolerate such large lateral deviation, a Laser Doppler Displacement Meter (Ref. 6) using a single aperture laser head and a flat-mirror as target can be used. This is because any lateral movement or movement perpendicular to the normal direction of the flat-mirror will not displace the laser beam. Hence the alignment is maintained. After 3 movements, the flat-mirror target will move back to the center of the diagonal again, hence the size of the flat-mirror has only to be larger than the largest increment.

In summary, in a conventional body diagonal measurement all 3 axes move simultaneously along a body diagonal and collect data at each preset increment. In the vector measurement all 3 axes move in sequence along a body diagonal and collect data after each axis is moved. Hence, not only 3 times more data are collected, the error due to the movement of each axis can also be separated.

## **IV.** Basic Theory

1) Trajectory Model

The general motion of a rigid body starting from point A and ending at point B can be described by 6 degrees of freedom. These are 1 linear position error, 2 straightness errors, and 3 angular errors as shown in Fig 2.



Fig. 1 A rigid body moved from A to B



To simplify the analysis, pick representative point Pa on the rigid body (such as the tool tip or probe tip), and move the coordinate such that at A, Pa is at the origin of the coordinate. Assume the motion is along the x-direction with an increment of X and move the origin of the new coordinate to X. If there is no error, Pb should be at the origin of the new coordinate. However, in general Pb is not at the origin. As shown in Fig. 3, in the new coordinate at B, Pb = X ux + E (x), (a bold letter indicates a vector quantity) where ux is the unit vector in the x-direction and E (x) is the vector position error (or volumetric error) due to the motion in x-direction. In general E (x) can be expressed as,

Where ux, uy, and uz are unit vectors in the directions of x-, y-, and zaxis, and Ex(x) is the error component in x-direction due to the motion in x, Ey(x) and Ez(x) are the error components in y- and z-direction respectively due to the motion in x. Please note that the Ex(x), Ey(x) and Ez(x) are the position error components due to all the motion errors including the position error, 2 straightness errors, 3 angular errors and even non-rigid body motion errors. Similarly, the errors due to y-axis motion and z-axis motion are E(y), and E(z) respectively, and can be expressed as,

$$\mathbf{E}(\mathbf{y}) = \mathbf{E}\mathbf{x}(\mathbf{y}) \mathbf{u}\mathbf{x} + \mathbf{E}\mathbf{y}(\mathbf{y}) \mathbf{u}\mathbf{y} + \mathbf{E}\mathbf{z}(\mathbf{y}) \mathbf{u}\mathbf{z}$$
  
Eq. 2  
$$\mathbf{E}(\mathbf{z}) = \mathbf{E}\mathbf{x}(\mathbf{z}) \mathbf{u}\mathbf{x} + \mathbf{E}\mathbf{y}(\mathbf{z}) \mathbf{u}\mathbf{y} + \mathbf{E}\mathbf{z}(\mathbf{z}) \mathbf{u}\mathbf{z}$$

### 2) Measurement Along Body Diagonal

Assume the measurement is along a diagonal direction  $\mathbf{R}$  with increments X, Y, and Z. The vector  $\mathbf{R}$  can be expressed as,

$$\mathbf{R} = X/R \mathbf{u}x + Y/R \mathbf{u}y + Z/R \mathbf{u}z$$
Eq. 3

The displacement error dR measured along the diagonal direction is the position error vector  $\mathbf{E}$  projected to the diagonal direction  $\mathbf{R}$ . Hence it is the scalar product of  $\mathbf{E}$  and  $\mathbf{R}$ . That is

$$d\mathbf{R} = \mathbf{E} \bullet \mathbf{R} = \mathbf{E}\mathbf{x}^*\mathbf{X}/\mathbf{R} + \mathbf{E}\mathbf{y}^*\mathbf{Y}/\mathbf{R} + \mathbf{E}\mathbf{z}^*\mathbf{Z}/\mathbf{R} \qquad \text{Eq. 4}$$

where • means a scalar product of two vectors.

More specifically,

$$dR(x) = Ex(x)*X/R + Ey(x)*Y/R + Ez(x)*Z/R$$
  
$$dR(y) = Ex(y)*X/R + Ey(y)*Y/R + Ez(y)*Z/R \qquad Eq. 5$$
  
$$dR(z) = Ex(z)*X/R + Ey(z)*Y/R + Ez(z)*Z/R$$

where dR(x) is the displacement error measured along the diagonal direction due to the movement of x-axis, dR(y) and dR(z) are the displacement errors measured along the diagonal direction due to the movement of y- and z-axis respectively.

#### 3) Measurement Along 4 Body Diagonals

There are 4 diagonals, namely,

from (0, 0, 0) to (nX, nY, nZ), denoted by ppp, from (nX, 0, 0) to (0, nY, nZ), denoted by npp, from (0, nY, 0) to (nX, 0, nZ), denoted by pnp, and from (0, 0, nZ) to (nX, nY, 0), denoted by ppn,

where n is the number of increments, ppp means all increments are positive, npp means all increments are positive except X, pnp means all increments are positive except Y, and ppn means all increments are positive except Z.

Let dR(x)(ppp) be the displacement error measured along the ppp diagonal direction due to the movement of x-axis. The first equation of Eq.5 becomes,

dR(x)(ppp) = Ex(x)\*X/R + Ey(x)\*Y/R + Ez(x)\*Z/R.

Similarly, for the other diagonals,

dR(x)(npp) = -Ex(x)\*X/R + Ey(x)\*Y/R + Ez(x)\*Z/R.

dR(x)(pnp) = Ex(x)\*X/R - Ey(x)\*Y/R + Ez(x)\*Z/R. Eq.6

$$dR(x)(ppn) = Ex(x)*X/R + Ey(x)*Y/R - Ez(x)*Z/R.$$

Solve Eq. 6 for Ex(x), Ey(x), and Ez(x), we have

$$Ex(x) = [dR(x)(ppp) - dR(x)(npp)] R/(2X)$$

Ey(x) = [dR(x)(ppp) - dR(x)(pnp)]\*R/(2Y) Eq. 7

Ez(x) = [dR(x)(ppp) - dR(x)(ppn)] \* R/(2Z)

Similarly for y-axis and z-axis movement,

Ex(y) = [dR(y)(ppp) - dR(y)(npp)]\*R/(2X) Ey(y) = [dR(y)(ppp) - dR(y)(pnp)]\*R/(2Y) Ez(y) = [dR(y)(ppp) - dR(y)(ppn)]\*R/(2Z) Ex(z) = [dR(z)(ppp) - dR(z)(npp)]\*R/(2X) Ey(z) = [dR(z)(ppp) - dR(z)(pnp)]\*R/(2Y) Ez(z) = [dR(z)(ppp) - dR(z)(pnp)]\*R/(2Z)

Substitute the measured displacement along all 4 diagonals, dR(ppp), dR(npp), dR(ppp), and dR(ppn), into Eq. 7 and 8, the position errors, Ex(x), Ey(x), Ez(x), Ex(y), Ey(y), Ez(y), Ex(z), Ey(z), and Ez(z) can be calculated.

#### 4) Error compensation

For most machine tools, the linear errors(sometimes called the pitch error or scale error) can be compensated by the controller. However, there are other errors such as straightness errors(guide way straightness) squareness errors (squareness between axes), angular errors (pitch, yaw and roll angles), and the non-rigid body errors (weight shifting, counter balancing etc). Usually, the straightness errors and the squareness errors are much larger than the linear errors, hence only compensate the linear errors is not enough.

Many controllers have the capability to do the cross compensations (sometimes called sag compensation). That is, compensate the errors in the ydirection and z-direction as a function of x, compensate the errors in the xdirection and z-direction as a function of y, and compensate the errors in the xdirection and y-direction as a function of z. These correspond to the volumetric error components, Ey(x), Ez(x), Ex(y), Ez(y), Ex(z), and Ey(z). Hence input the measured volumetric error components to the controller the straightness errors and the squareness errors can be compensated and reduce the machine tool positioning errors significantly.

## V. Experimental Verification

Using an Optodyne MCV-500 calibration laser system and a volumetric calibration package, the volumetric errors of a Giddings & Lewis, model RAM 630 horizontal machining center were measured. The measured volumetric errors, linear errors, vertical straightness, and horizontal straightness of X-axis, Y-axis, and Z-axis respectively are shown in Fig. 3a, 3b, and 3c respectively. Fig.3a shows that for the X-axis, the largest error is the linear error. Fig. 3b shows that for the Y-axis the largest error is the vertical straightness, which may be caused by the non- squareness. Fig. 3c shows that for the Z-axis, the largest error is the horizontal straightness. Hence, if the machine is compensated for displacement errors only, the large straightness errors in the Y-axis and in the Z-axis will not be compensated.

The volumetric errors of the machine were measured by the vector method without compensation. The measured volumetric errors Fig.3a, 3b, and 3c were used to generate the compensation files.



Fig. 3a A plot of linear and straightness errors due to x-movement.



Fig. 3b A plot of linear and straightness errors due to y-movement.



Fig. 3c A plot of linear and straightness errors due to z-movement.

The compensation files were loaded into the controller of the machine. The volumetric accuracy of the machine was checked by the conventional body diagonal measurement (ASME B5.54 standard). The bodies diagonal errors measured without compensation are plotted in Fig. 4a and the same measurements with compensation are plotted in Fig.4b. The diagonal errors without compensation are about 50 um and the diagonal errors with compensation are about 14 um. Hence an improvement of diagonal accuracy of a factor of 3 to 4 is achieved.







## Fig. 4b A plot of diagonal displacement measurements with compensation.

## VI. Summary and Conclusions

To achieve higher position accuracy of a machine tool, it is important to measure the volumetric errors and to compensate the volumetric errors provided that the machine tool is repeatable. Using the Optodyne MCV-500 laser calibration system and the vector method, the volumetric errors of a Giddings & Lewis, model, RAM 630 machining center have been measured. The measured volumetric errors were used to compensate the machine errors resulting in higher volumetric accuracy. The time required to compensate the machine using the vector method is significantly less than that using conventional measurement procedures.

In summary, we have shown that the volumetric errors of a machine tool can easily be measured by the vector measurement technique developed by Optodyne. The measured volumetric errors can be used to compensate the machine errors and achieve higher volumetric accuracy. Furthermore, the time required to compensate the machine using the vector method is significantly less than that using conventional measurement procedures making it easier for manufactures to take advantage to the advancements in CNC control technology.

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## **Figure Captions**

- **1.** A rigid body motion from point A to point B.
- 2. Volumetric displacement errors or a vector error.
- **3.** Plots of linear and straightness errors.
- **4.** Plots of conventional body diagonal measurement with and without volumetric compensation.