A theoretical analysis of 4 body diagonal displacement measurement and sequential step diagonal measurement

Charles Wang and Gianmarco Liotto
Optodyne, Inc.,
Compton, CA 90220, U.S.A.
Email: Optodyne@aol.com

Abstract

The introduction of B5.54 and ISO230-6 machine tool performance measurement standards are increasing the popularity of laser interferometer diagonal, sequential step diagonal or vector technique for the calibration and compensation of machine tool errors.

Reported here is a theoretical analysis of the 4 body diagonal displacement measurement, sequential step diagonal or vector technique, and 3 axes linear displacement measurement. The relations between the 21 rigid body machine positioning errors and the 4 body diagonal displacement measurement; and the sequential step diagonal measurement; and the 3 axes linear displacement measurement were derived. Based on these relations, the advantages and limitations of these measurements have been analyzed.

1. Introduction

The measurement and compensation of 3D volumetric positioning accuracy becomes more important for quality assurance and to achieve higher accuracy. The introduction of B5.54 and ISO230-6 machine tool performance measurement standards [1,2] are increasing the popularity of laser interferometer diagonal, sequential step diagonal or vector technique [3,4] for the calibration and compensation of machine tool errors. The B5.54 body diagonal displacement tests have been used by Boeing Aircraft Company and many others for many years with very good results and success. This is due to the reduction in calibration time these methods can provide over the conventional laser interferometer based measurement taken along lines parallel to the machine's X, Y and Z axes.

Here a theoretical analysis on the relations between the 21 rigid body positioning errors and the 4 body diagonal displacement measurement; and the sequential step diagonal displacement errors; and the 3 axis linear displacement measurement; has been performed. It is concluded that the 4 body diagonal displacement measurement can be used for a quick check on the volumetric positioning accuracy. The sequential step diagonal measurement can be used to determine the volumetric positioning errors, 3 displacement errors, 6 straightness errors and 3 squareness errors. The 3 axis linear displacement measurement can be used to determine the 3 displacement errors, 3 pitch angular errors and 3 yaw angular errors.

Using these measurements, all 21 rigid body positioning errors, except roll angular errors, can all be measured. The measurement accuracy is limited by the repeatability of the machine. The performance of these measurements is relatively simple and straight forward. A single laser interferometer with linear displacement capability can be used to perform all these measurement. Now, manufacturing can migrate from one-dimensional linear to 3D laser volumetric measurement without incurring high costs and long machine tool down time.

2. Positioning errors of a 3-axis machine

For a 3-axis machine, there are 6 errors per axis or a total of 18 errors plus 3 squareness errors. These 21 rigid body errors [5] can be expressed as the following.

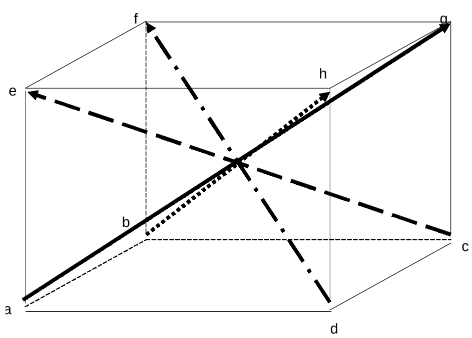
Linear displacement errors: Dx(x), Dy(y), and Dz(z)
Vertical straightness errors: Dy(x), Dx(y), and Dx(z)
Horizontal straightness errors: Dz(x), Dz(y), and Dy(z)
Roll angular errors: Ax(x), Ay(y), and Az(z)
Pitch angular errors: Ay(x),Ax(y), and Ax(z)
Yaw angular errors: Az(x), Az(y), and Ay(z)
Squareness errors: Øxy, Øyz, Øzx,

where, D is the linear error, subscript is the error direction and the position coordinate is inside the parenthesis, A is the angular error, subscript is the axis of rotation and the position coordinate is inside the parenthesis.

Machine working volume: x = 0 to X, y = 0 to Y, and z = 0 to Z.
Measurement increment: a for x-axis, b for y-axis, c for z-axis, and r for body diagonal, where $r^2 = a^2 + b^2 + c^2$.
Number of measurement points: N + 1 (including 0) points, or X = Na, Y = Nb, Z = Nc and R = Nr.
Machine configuration: FXYZ

3. Formulae for 4 body diagonal displacement errors

For the 4 body diagonal displacement measurement [1,2], the measurement directions are ag. bh, ce, and df, as shown in Figure 1. The measurement is performed with the laser pointing along the body direction and the retroreflector moving along the body diagonal with a fixed increment as shown in Figure 2.



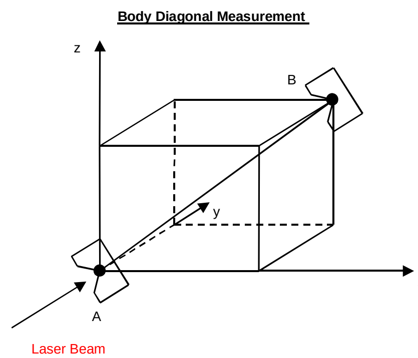


Figure 1: Shows the 4 body diagonal directions. Figure 2: Shows the laser beam direction and the retroreflector.

The measured error DR at each increment can be expressed as:

$$
\begin{aligned}
DRppp = {} & a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) \\
& + a/r*[Dx(y) + y/Y\ Øxy] + b/r*Dy(y) + c/r*Dz(y) \\
& + a/r*[Dx(z) + z/Z\ Øzx] + b/r*[Dy(z) + z/Z\ Øyz] + c/r * Dz(z) \\
& + Ay(x)*ac/r - Az(x)*ab/r + Ay(y)*ac/r - Ax(y)*bc/r.
\end{aligned} \qquad (1)
$$

$$
\begin{aligned}
DRnpp = {} & -a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) + \\
& - a/r*[Dx(y) + y/Y\ Øxy] + b/r*Dy(y) + c/r*Dz(y) \\
& - a/r*[Dx(z) + z/Z\ Øzx] + b/r*[Dy(z) + z/Z\ Øyz] + c/r * Dz(z) \\
& - Ay(x)*ac/r + Az(x)*ab/r - Ay(y)*ac/r - Ax(y)*bc/r.
\end{aligned} \qquad (2)
$$

$$
\begin{aligned}
DRpnp = {} & a/r * Dx(x) - b/r * Dy(x) + c/r * Dz(x) \\
& + a/r*[Dx(y) + y/Y\ Øxy] - b/r*Dy(y) + c/r*Dz(y) \\
& + a/r*[Dx(z) + z/Z\ Øzx] - b/r*[Dy(z) + z/Z\ Øyz] + c/r * Dz(z) \\
& + Ay(x)*ac/r + Az(x)*ab/r + Ay(y)*ac/r + Ax(y)*bc/r.
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
DRppn = {} & a/r * Dx(x) + b/r * Dy(x) - c/r * Dz(x) \\
& + a/r*[Dx(y) + y/Y\ Øxy] + b/r*Dy(y) - c/r*Dz(y) \\
& + a/r*[Dx(z) + z/Z\ Øzx] + b/r*[Dy(z) + z/Z\ Øyz] - c/r * Dz(z) \\
& - Ay(x)*ac/r - Az(x)*ab/r - Ay(y)*ac/r + Ax(y)*bc/r.
\end{aligned} \tag{4}
$$

where the subscript ppp means body diagonal with all x, y and z positive; npp means body diagonal with x negative, y and z positive; pnp means body diagonal with y negative, x and z positive; and ppn means body diagonal with z negative, x and y positive.

In general, the positioning errors generated by the angular errors are all in the direction perpendicular to the Abbe offset direction or the body diagonal direction. Hence the displacement errors in the body diagonal directions are not sensitive to any angular errors. However, for other configurations, such as XFYZ and XYFZ, the angular terms are $Ay(y)*ac/r - Ax(y)*bc/r$ and $Ax(x)*bc/r - Ay(x)*ac/r + Az(x)*ab/r$ respectively.

The 4 body diagonal displacement errors shown in Eqns. 1 to 4 are sensitive to all of the 9 linear errors. Hence it is a good measurement of the volumetric positioning errors, namely, 3 displacement errors, 6 straightness errors and 3 squareness errors.

The errors in the above equations may be positive or negative. Hence, they may cancel each other. However, the errors are statistical in nature, the probability that all of the errors will be cancelled in all of the positions and in all of the 4 body diagonals are theoretically possible but very unlikely. Hence it is indeed a quick measurement of volumetric positioning accuracy. We can conclude that if the 4 body diagonal displacement errors are small, then the machine errors are most likely very small. If the 4 body diagonal displacement errors are large, then we can conclude that the machine errors are large. However, because there are only 4 sets of data and there are 9 sets of errors, we do not have enough information to determine which errors are large.

4. Formulae for 4 sequential step diagonal errors

To overcome the limitations in the 4 body diagonal displacement measurement, a sequential step diagonal or vector technique [3] has been developed by Optodyne. As shown in Figure 3, instead of moving along the body diagonal direction, move x-axis first, stop and collect data, then move y-axis, stop and collect data, then move z-axis, stop and collect data. Repeat the same until reached the other corner of the body diagonal. Since the laser beam direction and moving direction are not the same, a flat-mirror has to be used to replace the

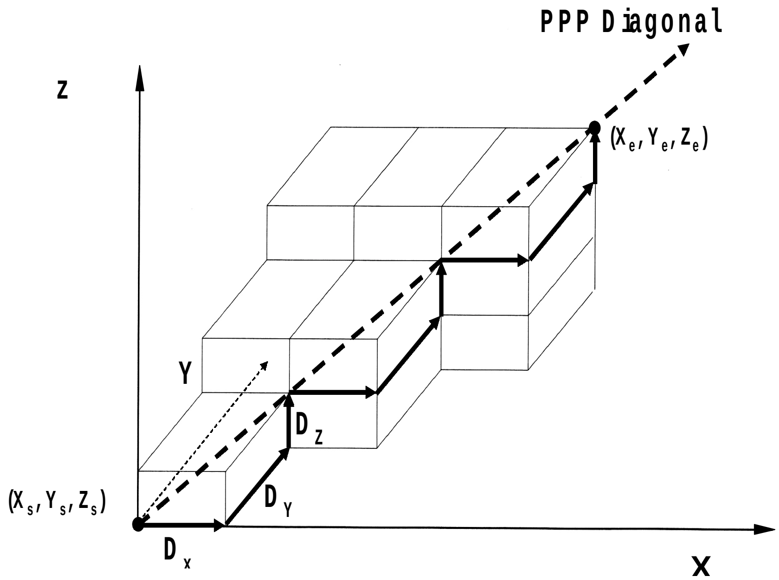


Figure 3: The vector measurement, laser is pointing in the ppp diagonal direction and the sequence is moving x-axis, stop, collect data, moving y-axis, stop, collect data, moving z-axis, stop, collect data and continue.

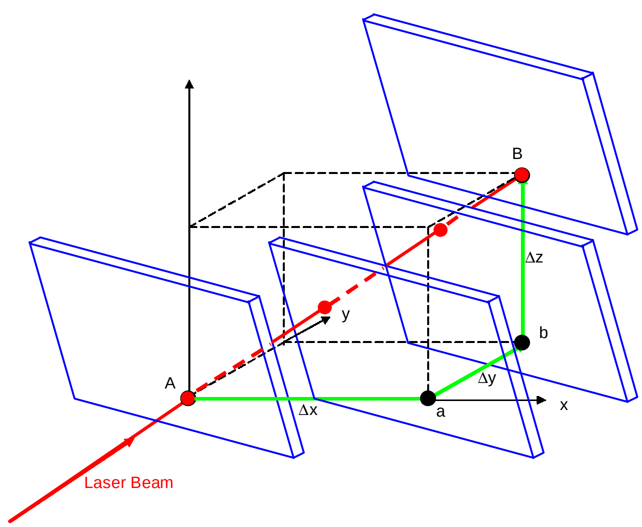


Figure 4: Shows the sequential step diagonal measurement using a flat-mirror as Target

retroreflector target as shown in Figure 4. Hence, for each body diagonal measurement, because of the sequential steps, there are 3 sets of data. For 4 body diagonal measurement, there are 12 sets of data. Using these 12 sets of data we can solve the 3 displacement errors, 6 straightness errors and 3 squareness errors. More detailed theory is in Ref. [3] and the experimental verification is in Ref. [4, 6].

In the 4 sequential step diagonal or vector measurement, the measured error DR at each sequential step DR(x), DR(y), and DR(z) and at each increment can be expressed as:

$$DRppp(x) = a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) + f * (c\,Ay(x) - b\,Az(x)), \quad (5)$$

$$DRppp(y) = a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) + c/r*Dz(y) + g*(a\,Ay(y) - c\,Ax(y)), \quad (6)$$

$$DRppp(z) = a/r*[Dx(z) + z/Z\,Øzx] + b/r*[Dy(z) + z/Z\,Øyz] + c/r * Dz(z) - f * (c\,Ay(x) - b\,Az(x)) - g*(a\,Ay(y) - c\,Ax(y)), \quad (7)$$

$$DRnpp(x) = -a/r * Dx(x) + b/r * Dy(x) + c/r * Dz(x) + f * (c\,Ay(x) - b\,Az(x)), \quad (8)$$

$$DRnpp(y) = -\,a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) + c/r*Dz(y) + g*(-a\,Ay(y) - c\,Ax(y)), \quad (9)$$

$$DRnpp(z) = -\,a/r*[Dx(z) + z/Z\,Øzx] + b/r*[Dy(z) + z/Z\,Øyz] + c/r * Dz(z) - f * (c\,Ay(x) - b\,Az(x)) - g*(-a\,Ay(y) - c\,Ax(y)), \quad (10)$$

$$DRpnp(x) = a/r * Dx(x) - b/r * Dy(x) + c/r * Dz(x) + f * (c\,Ay(x) + b\,Az(x)), \quad (11)$$

$$DRpnp(y) = +\,a/r*[Dx(y) + y/Y\,Øxy] - b/r*Dy(y) + c/r*Dz(y) + g*(a\,Ay(y) - c\,Ax(y)), \quad (12)$$

$$DRpnp(z) = +\,a/r*[Dx(z) + z/Z\,Øzx] - b/r*[Dy(z) + z/Z\,Øyz] + c/r * Dz(z) - f * (c\,Ay(x) + b\,Az(x)) - g*(a\,Ay(y) - c\,Ax(y)), \quad (13)$$

$$DRppn(x) = a/r * Dx(x) + b/r * Dy(x) - c/r * Dz(x) + f * (-c\,Ay(x) - b\,Az(x)), \quad (14)$$

$$DRppn(y) = +\,a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) - c/r*Dz(y) + g*(a\,Ay(y) + c\,Ax(y)), \quad (15)$$

$$DRppn(z) = +\,a/r*[Dx(z) + z/Z\,Øzx] + b/r*[Dy(z) + z/Z\,Øyz] - c/r * Dz(z) - f * (-c\,Ay(x) - b\,Az(x)) - g*(a\,Ay(y) + c\,Ax(y)), \quad (16)$$

Where the geometric factors $f^2 = (4a^2 + b^2 + c^2)/(b^2 + c^2)$ and $g^2 = (4b^2 + a^2 + c^2)/(a^2 + c^2)$.

The formulae for each set of data are shown in the above 12 equations (Eqns. 5 to 16). However, there are 13 unknowns, Dx(x), Dy(y), Dz(z), Dy(x), Dx(y), Dx(z), Dz(x), Dz(y), Dy(z), Ay(x), Az(x), Ax(y) and Ay(y). Assuming the roll angular error Ay(y) and the mirror misalignment errors are small, the 12 equations with 12 unknowns can be solved. The mirror misalignment errors can be minimized by either using a large retroreflector and flat-mirror combination or carefully centering the return beam to the aperture. If assuming all the angular errors are small, there are 9 unknowns and the above equations can be solved even with large mirror misalignment errors [3]. If all the angular errors are not small, additional measurement are needed. This can be achieved by perform 3 linear displacement measurements with different Abbe offsets on each axis such as described in the section 6 below. Once all the angular errors (except roll errors) are determined, the above 12 sets of equations can be used to solve the 9 linear errors with redundancy. For the experimental verification of the theory, see Ref [4, 6].

5. Formulae for 2 face diagonal displacement errors

The 2 face diagonal displacement measurement in the xy-plane is a special case of the 4 body diagonal displacement measurement, the measured error DR at each increment can be expressed as:

$$DRpp = a/r * Dx(x) + b/r * Dy(x) + a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) – Az(x)*ab/r \qquad (17)$$

$$DRnp = -a/r * Dx(x) + b/r * Dy(x) - a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) + Az(x)*ab/r \qquad (18)$$

Similarly for the yz- and xz-plane.

For the sequential step face diagonal measurement in the xy-plane, the measured errors DR(x) and DR(y) at each increment can be expressed as:

$$DRpp(x) = a/r * Dx(x) + b/r * Dy(x) - h * b\,Az(x)), \qquad (19)$$
$$DRpp(y) = a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) + h * b\,Az(x)), \qquad (20)$$
$$DRnp(x) = -a/r * Dx(x) + b/r * Dy(x) - h * b\,Az(x)), \qquad (21)$$
$$DRnp(y) = - a/r*[Dx(y) + y/Y\,Øxy] + b/r*Dy(y) + h * b\,Az(x)). \qquad (22)$$

where the geometric factor $h^2 = (4a^2 + b^2)/b^2$.

Similarly for the yz- and xz-plane. The formula for each set of data are shown in the above 4 equations (Eqns. 19 and 22). However, there are 5 unknowns, Dx(x), Dy(y), Dy(x), Dx(y), and Az(x). Assuming the angular error Az(x) is small, the 4 equations with 4 unknowns can be solved exactly.

6. Formulae for 3 axis linear displacement errors

For the linear displacement error measurement, the measured error D along x, y, and z-axis at each increment can be expressed as:

$$DX = Dx(x) + m * Ay(x) + p * Az(x), \qquad (23)$$
$$DY = Dy(y) + q * Ax(y) + s * Az(y), \qquad (24)$$
$$DZ = Dz(z) + t * Ax(z) + u * Ay(z), \qquad (25)$$

where Abbe offsets m and p are distances from the measurement line to the center line in y and z directions respectively, q and s are distances from the measurement line to the center line in x and z directions respectively, t and u are distances from the measurement line to the center line in x and y directions respectively.

Please note that the measured linear displacement errors are not the same, if not measured from the center line of the work volume. Also, the measured linear displacement errors are not sensitive to the straightness errors, hence it cannot be used to determine the straightness errors.

However, the linear displacement measurement can be used to determine the pitch and yaw angular errors by making 3 measurements, one along the top edge, one along the bottom edge, and one along the side edge of the working volume. The differences in the two measurements along the vertical edges (top and bottom) divided by the Abbe offset is the pitch angular error, and the differences in the two measurements along the horizontal edges (left and right) divided by the Abbe offset is the yaw angular error.

For example, for 3 measurements along X-axis at 3 different locations with known Abbe offsets m1,p1; m2, p2; and m3, p3, the Eq. 23 becomes,

$$DX1 = Dx(x) + m1 * Ay(x) + p1 * Az(x),$$
$$DX2 = Dx(x) + m2 * Ay(x) + p2 * Az(x),$$
$$DX3 = Dx(x) + m3 * Ay(x) + p3 * Az(x),$$

There are 3 sets of data DX1, DX2 and DX3 and 3 unknowns Dx(x), Ay(x) and Az(x). The solutions are,

$$Ay(x) = [(m3-m1)*(DX2-DX1)-(m2-m1)*(DX3-DX1)] / [(m3-m1)*(p2-p1) -(m2-m1)*(p3-p1)].$$
$$Az(x) = [(p3-p1)*(DX2-DX1)-(p2-p1)*(DX3-DX1)] / [(m3-m1)*(p2-p1) -(m2-m1)*(p3-p1)].$$
$$Dx(x) = DX1*(m2*p3-m3*p2) + DX2*(m3*p1 - m1*p3) + DX3*(m1*p2 - m2*p1) / [(m3-m1)*(p2-p1)-(m2-m1)*(p3-p1)].$$

Similarly for the Y- and Z-axis, Ax(y), Az(y), Dy(y), Ax(z), Ay(z), and Dz(z) can all be determined.

Of course, the accuracy of the measurement is limited by the repeatability of the machine and the Abbe offset. For example, for a machine with repeatability of 0.0001"(2.5 µm) and the Abbe offset of 20"(500 mm), the accuracy of the angular measurement is 0.0001/20 = 0.000005 rad. or 1 arcsec which is good enough for most of the machines.

7. Summary and conclusions

In summary, the 4 body diagonal displacement measurement is a quick measure of all the 9 linear errors, 3 displacement errors, 6 straightness errors and 3 squareness errors. The sequential step diagonal measurement provides 12 sets of measurement data to solve for all the 9 linear errors and 3 squareness errors. With additional 3 sets of linear displacement measurement for each axis, all the angular errors (except roll) can be determined.

In conclusion, the 4 body diagonal displacement measurement can be used for a quick check of the volumetric positioning accuracy. The sequential body diagonal or vector technique can be used to determine the volumetric positioning errors, 3 displacement errors, 6 straightness errors and 3 squareness errors, useful for error compensation. The 3 axis linear displacement measurement performed along 3 edges of the working volume can be used to determine the pitch and yaw angular errors.

It is noted that a single laser interferometer with linear displacement measurement capability can be used to perform all these measurements. Of course, the accuracy of all these measurements is limited by the repeatability of the machine. These measurement techniques will help the manufacturing industry to migrate from one-dimensional linear to 3D volumetric measurement without incurring high costs and long machine tool down time.

References

[1] Methods for Performance Evaluation of Computer Numerically Controlled Machining Centers, *An American National Standard*, ASME B5.54-1992 by the American Society of Mechanical Engineers, p69, 1992.

[2] ISO 230-6: 2002 Test code for machine tools – Part 6: Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests)", *an International Standard*, by International Standards Organization, 2002.

[3] Wang, C., Laser Vector measurement Technique for the determination and compensation of volumetric positioning errors. Part I: Basic theory, *Review of Scientific Instruments*, Vol. 71, No 10, pp 3933-3937, 2000.

[4] Jenecko, J., Griffin, B., and Wang, C., Laser Vector Measurement Technique for the determination and compensation of volumetric positioning errors. Part II: Experimental verification, *Review of Scientific Instruments*, Vol. 71, No. 10, pp.3938-3941, 2000.

[5] Schultschik, R., The components of the volumetric accuracy, *Annals of the CIRP*, Vol.25, No.1, pp223-228, 1977.

[6] Wang, C. and Liotto, G., A laser non-contact measurement of static positioning and dynamic contouring accuracy of a CNC machine tool, *Proceedings of the Measurement Science Conference*, Los Angeles, January 24-25, 2002.

LAMDAMAP03cpwangR.DOC
Revised 7/4//2003