Theoretical derivations of 4 body diagonal displacement errors in 4 machine configurations

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Abstract

The measurement of 21 rigid body errors is very difficult and time consuming. The introduction of B5.54 and ISO230-6 machine tool performance measurement standards are increasing the popularity of laser interferometer diagonal displacement measurement for the calibration of machine tool positioning errors. To establish the theoretical foundation of the 4 body diagonal displacement measurement, it is important to derive the relations of the 21 rigid body errors and the measured 4 body diagonal displacement errors.

To derive these relations, there are three different approaches. One is manually calculate the 3rd order vector and rotation matrices, the 2nd is using the MathLab program to calculate the 4th order translation and rotation matrices and the third is using a stacking model and Abbe offsets to determine the angular error terms. Using these three approaches, the formulae for the 4 body diagonal displacement errors in 4 machine configurations, FXYZ, XFYZ, XYFZ, and XYZ have been derived. As expected, the results derived by all three approaches are the same. The results show that for the XFYZ and XYFZ configurations most of the angular errors are cancelled and only 2 angular error terms are left in the body diagonal displacement measurement.

1. Introduction

The worldwide competition and quality standards, such as ISO 9000 and QS 9000, all demand tighter tolerance and regular maintenance of all machine tools. To generate good quality or accurate parts, the measurement of 3 dimensional volumetric positioning accuracy of a machine tool is critical. The introduction of B5.54 [1] and ISO230-6 [2] machine tool performance measurement standards are increasing the popularity of laser interferometer diagonal, sequential step diagonal or vector technique[3] for the calibration and compensation of machine tool errors.

Using a conventional laser interferometer to measure the straightness and squareness errors is rather difficult and costly. It usually takes days of machine down time and experienced operator to perform these measurements. For those reasons the body diagonal displacement error defined in the ASME B5.54 or ISO 230-6 standard is a good quick check of the volumetric error. Furthermore, it has been used by Boeing Aircraft Company and many others for many years with very good results and success. However, it is not clear, what is the relation between the body diagonal displacement errors and the 3D positioning errors.

2. Positioning errors of 3-axis machines and machine configurations

For a 3-axis machine, there are 6 errors per axis or a total of 18 errors plus 3 squareness errors. These 21 rigid body errors [4] can be expressed as the following.

Linear displacement errors: $\delta_x(x)$, $\delta_y(y)$ and $\delta_z(z)$
Vertical straightness errors: $\delta_y(x)$, $\delta_x(y)$ and $\delta_x(z)$
Horizontal straightness errors: $\delta_z(x)$, $\delta_z(y)$ and $\delta_y(z)$
Roll angular errors: $\varepsilon_x(x)$, $\varepsilon_y(y)$ and $\varepsilon_z(z)$
Pitch angular errors: $\varepsilon_y(x)$, $\varepsilon_x(y)$ and $\varepsilon_x(z)$
Yaw angular errors: $\varepsilon_z(x)$, $\varepsilon_z(y)$ and $\varepsilon_y(z)$
Squareness errors: S_{xy}, S_{yz}, S_{zx}
where, x, y, z are the coordinates, δ is the linear error, subscript is the error direction and the position coordinate is inside the parenthesis, ε is the angular error, subscript is the axis of rotation and the position coordinate is inside the parenthesis.

In most cases coordinate measuring machines and machine tools can be classified into four configurations[5]. They are the FXYZ, XFYZ, XYFZ, and XYZF as shown in Figures 1, 2, 3, and 4 respectively. Here, the axis before F show available motion directions of the work piece with respect to the base, and the letters after F show the available motion directions of the tool (or probe) with respect to the base. For example, in FXYZ the work piece is fixed, and in XYZF the tool is fixed.

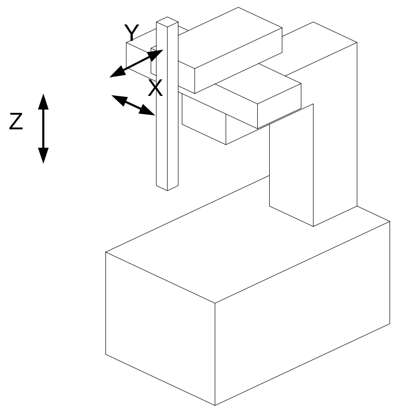

Fig. 1, Schematic of FXYZ

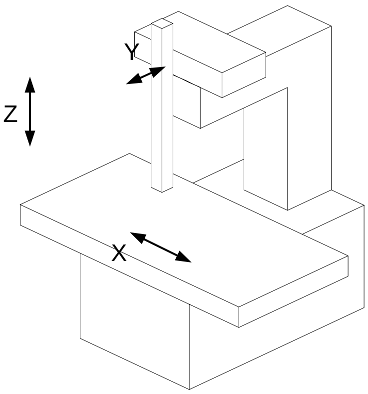

Fig. 2 Schematic of XFYZ

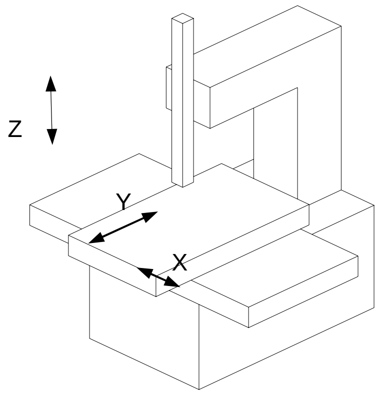

Fig. 3, Schematic of XYFZ

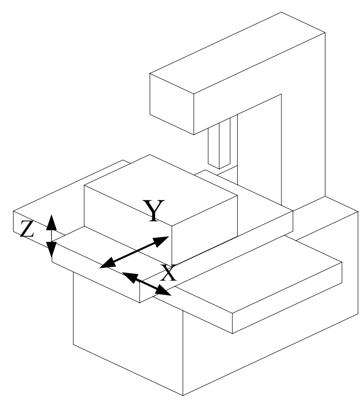

Fig. 4, Schematic of XYZF

3. Position vector and rotation matrix

The vector positions of each stage, **X, Y** and **Z** can be expressed as column vectors,

$$\mathbf{X} = \begin{bmatrix} x + \delta_{\mathrm{x}}(x) \\ \delta_{\mathrm{y}}(x) \\ \delta_{\mathrm{z}}(x) \end{bmatrix} \tag{1}$$

$$\mathbf{Y} = \begin{bmatrix} \delta_x(y) \\ y + \delta_y(y) \\ \delta_z(y) \end{bmatrix} \quad (2)$$

$$\mathbf{Z} = \begin{bmatrix} \delta_x(z) \\ \delta_y(z) \\ z + \delta_z(z) \end{bmatrix} \quad (3)$$

To simplify the calculation, the squareness errors can be included in the straightness errors by define the new straightness error as the sum of the old straightness errors and the squareness errors as shown below.

$$\begin{aligned} \delta_x(y) &= \delta_x(y)\ (\text{old}) + S_{xy} * y \\ \delta_x(z) &= \delta_x(z)\ (\text{old}) + S_{zx} * z \\ \delta_y(z) &= \delta_y(z)\ (\text{old}) + S_{yz} * z \end{aligned} \quad (4)$$

To simplify the calculation, neglect the tool offset.
The angular error rotation matrix can be expressed as,

$$\mathbf{R}(u) = \begin{bmatrix} 1 & \varepsilon_z(u) & -\varepsilon_y(u) \\ -\varepsilon_z(u) & 1 & \varepsilon_x(u) \\ \varepsilon_y(u) & -\varepsilon_x(u) & 1 \end{bmatrix} \quad (5)$$

where u = x, y or z.

Please note $\varepsilon_u(u)$ is much smaller than 1 and also an odd function of u, hence $\mathbf{R}(u)\ \mathbf{I} = \mathbf{I}\ \mathbf{R}(u)$, and $\mathbf{R}(-u) = \mathbf{R}^{-1}(u)$, where **I** is a unit matrix and $\mathbf{R}^{-1}$ is the inverse matrix of **R**.

4. 3rd order vectors and rotation matrices calculation

If the positions of the X, Y, and Z stages are represented by the vectors **X, Y,** and **Z** respectively. The angular errors of the X, Y, and Z stages are represented by the rotation matrices **R**(x), **R**(y), and **R**(z). The actual positions with respect to the work piece or machine coordinate can be represented by the vector **P**. As shown in [5], the actual position vector **P** for the 4 configurations can be expressed in a machine coordinate as the followings.

For FXYZ: $\mathbf{P} = \mathbf{X} + \mathbf{R}^{-1}(x)\mathbf{Y} + \mathbf{R}^{-1}(x)\mathbf{R}^{-1}(y)\mathbf{Z}$ (6)

For XFYZ: $\mathbf{P} = \mathbf{R}^{-1}(x)\mathbf{X} + \mathbf{R}^{-1}(x)\mathbf{Y} + \mathbf{R}^{-1}(x)\ \mathbf{R}^{-1}(y)\mathbf{Z}$ (7)

For XYFZ: $\mathbf{P} = \mathbf{R}^{-1}(y)\ \mathbf{R}^{-1}(x)\mathbf{X} + \mathbf{R}^{-1}(y)\mathbf{Y} + \mathbf{R}^{-1}(y)\ \mathbf{R}^{-1}(x)\mathbf{Z}$ (8)

For XYZF: $\mathbf{P} = \mathbf{R}^{-1}(z)\ \mathbf{R}^{-1}(y)\ \mathbf{R}^{-1}(x)\mathbf{X} + \mathbf{R}^{-1}(z)\ \mathbf{R}^{-1}(y)\mathbf{Y} + \mathbf{R}^{-1}(z)\mathbf{Z}$ (9)

The actual tool tip position can be expressed as a column vector,

$$\mathbf{P} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \quad (10)$$

For FXYZ configuration, Eqn. (6) becomes

$$\begin{aligned} P_x - x &= [\delta_x(x) - y*\varepsilon_z(x) + z*\varepsilon_y(x)] + [\delta_x(y) + z*\varepsilon_y(y)] + [\delta_x(z)] \\ P_y - y &= [\delta_y(x) - z*\varepsilon_x(x)] + [\delta_y(y) - z*\varepsilon_x(y)] + [\delta_y(z)] \\ P_z - z &= [\delta_z(x) + y*\varepsilon_x(x)] + [\delta_z(y)] + [\delta_z(z)] \end{aligned} \quad (11)$$

Similarly for XFYZ configuration, Eqn. (7) becomes

$$\begin{aligned} P_x - x &= [\delta_x(x) - y*\varepsilon_z(x) + z*\varepsilon_y(x)] + [\delta_x(y) + z*\varepsilon_y(y)] + [\delta_x(z)] \\ P_y - y &= [\delta_y(x) + x*\varepsilon_z(x) - z*\varepsilon_x(x)] + [\delta_y(y) - z*\varepsilon_x(y)] + [\delta_y(z)] \\ P_z - z &= [\delta_z(x) - x*\varepsilon_y(x) + y*\varepsilon_x(x)] + [\delta_z(y)] + [\delta_z(z)] \end{aligned} \quad (12)$$

Similarly for XYFZ configuration, Eqn (8) becomes

$$\begin{aligned} P_x - x &= [\delta_x(x) + z*\varepsilon_y(x)] + [\delta_x(y) - y*\varepsilon_z(y) + z*\varepsilon_y(y)] + [\delta_x(z)] \\ P_y - y &= [\delta_y(x) + x*\varepsilon_z(x) - z*\varepsilon_x(x)] + [\delta_y(y) + x*\varepsilon_z(y) - z*\varepsilon_x(y)] + [\delta_y(z)] \\ P_z - z &= [\delta_z(x) - x*\varepsilon_y(x)] + [\delta_z(y) - x*\varepsilon_y(y) + y*\varepsilon_x(y)] + [\delta_z(z)] \end{aligned} \quad (13)$$

Finally for XYZF configuration, Eqn. (9) becomes,

$$\begin{aligned} P_x - x &= [\delta_x(x)] + [\delta_x(y) - y*\varepsilon_z(y)] + [\delta_x(z) - y*\varepsilon_z(z) + z*\varepsilon_y(z)] \\ P_y - y &= [\delta_y(x) + x*\varepsilon_z(x)] + [\delta_y(y) + x*\varepsilon_z(y)] + [\delta_y(z) + x*\varepsilon_z(z) - z*\varepsilon_x(z)] \\ P_z - z &= [\delta_z(x) - x*\varepsilon_y(x)] + [\delta_z(y) - x*\varepsilon_y(y) + y*\varepsilon_x(y)] + [\delta_z(z) - x*\varepsilon_y(z) + y*\varepsilon_x(z)] \end{aligned} \quad (14)$$

The tool offset errors are the same for all 4 configurations.

5. Forth order transformation matrices calculation

The second approach is using the forth order transformation matrices formulation and the MathLab program [6].

For FXYZ configuration machine, setup coordinate systems X、Y、Z for X、Y、Z Stage respectively, coordinate system τ for tool tip and p for workpiece. Finally setup a reference system R at the machine bed, and assume the origins of the coordinate systems coincide at the same point before running. Under ideal conditions(all the errors are zero), the homogeneous coordinate transformation matrices can be expressed as:

$$^{i}\mathbf{T}_R^X = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^{i}\mathbf{T}_X^Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^{i}\mathbf{T}_Y^Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{i}\mathbf{T}_Z^\tau = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^{i}\mathbf{T}_p^R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

So in ideal case, the transformation matrices between coordinate systems τ and p can be expressed as:

$$ {}^{\mathrm{i}}\mathrm{T}_{p}^{\tau} = {}^{\mathrm{i}}\mathrm{T}_{p}^{R} \cdot {}^{\mathrm{i}}\mathrm{T}_{R}^{\tau} = {}^{\mathrm{i}}\mathrm{T}_{p}^{R} \cdot ({}^{\mathrm{i}}\mathrm{T}_{R}^{X} \cdot {}^{\mathrm{i}}\mathrm{T}_{X}^{Y} \cdot {}^{\mathrm{i}}\mathrm{T}_{Y}^{Z} \cdot {}^{\mathrm{i}}\mathrm{T}_{Z}^{\tau} \cdot {}^{\mathrm{i}}\mathrm{T}_{Z}^{\tau}) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (16) $$

where the superscript i represents the ideal condition.

Under error condition, there are three translational and three rotational error components for X 、Y、Z stage, using rigid body kinematics with small errors approximation and homogeneous coordinate transformation, the homogeneous coordinate transformation matrices can obtained as:

$$ {}^{e}\mathbf{T}_{R}^{X} == \begin{bmatrix} 1 & -\varepsilon_{z}(x) & \varepsilon_{y}(x) & x+\delta_{x}(x) \\ \varepsilon_{z}(x) & 1 & -\varepsilon_{x}(x) & \delta_{y}(x) \\ -\varepsilon_{y}(x) & \varepsilon_{x}(x) & 1 & \delta_{z}(x) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{e}\mathbf{T}_{X}^{Y} == \begin{bmatrix} 1 & -\varepsilon_{z}(y) & \varepsilon_{y}(y) & \delta_{x}(y) \\ \varepsilon_{z}(y) & 1 & -\varepsilon_{x}(y) & \mathrm{y}+\delta_{y}(y) \\ -\varepsilon_{y}(y) & \varepsilon_{x}(y) & 1 & \delta_{z}(y) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (17) $$

$$ {}^{e}\mathbf{T}_{R}^{X} == \begin{bmatrix} 1 & -\varepsilon_{z}(z) & \varepsilon_{y}(z) & \delta_{x}(z) \\ \varepsilon_{z}(z) & 1 & -\varepsilon_{x}(z) & \delta_{y}(z) \\ -\varepsilon_{y}(z) & \varepsilon_{x}(z) & 1 & \mathrm{z}+\delta_{z}(z) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{e}\mathbf{T}_{Z}^{\tau} = {}^{i}\mathbf{T}_{Z}^{\tau} = \mathbf{I}, \quad {}^{e}\mathbf{T}_{p}^{R} = {}^{i}\mathbf{T}_{p}^{R} = \mathbf{I} $$

where the superscript e represents the error case and **I** is a unit matrix.

Hence under the error case, the transformation matrices between coordinate systems τ and p can be expressed as:

$$ {}^{\mathrm{e}}\mathrm{T}_{p}^{\tau} = {}^{\mathrm{e}}\mathrm{T}_{p}^{R} \cdot {}^{\mathrm{e}}\mathrm{T}_{R}^{\tau} = {}^{\mathrm{e}}\mathrm{T}_{p}^{R} \cdot ({}^{\mathrm{e}}\mathrm{T}_{R}^{X} \cdot {}^{\mathrm{e}}\mathrm{T}_{X}^{Y} \cdot {}^{\mathrm{e}}\mathrm{T}_{Y}^{Z} \cdot {}^{\mathrm{e}}\mathrm{T}_{Z}^{\tau} \cdot {}^{\mathrm{e}}\mathrm{T}_{Z}^{\tau}) \qquad (18) $$

$$ {}_{p}^{\tau}\boldsymbol{T}^{\mathrm{e}} = {}_{p}^{\tau}\boldsymbol{E} \cdot {}_{p}^{\tau}\boldsymbol{T}^{\mathrm{i}} \qquad (19) $$

where error matrix ${}_{\tau}^{p}\boldsymbol{E}$ between coordinate systems τ and p can be expressed as

$$ {}^{p}_{\tau}\boldsymbol{E} = \begin{bmatrix} 1 & -\gamma_z & \gamma_y & \eta_x \\ \gamma_z & 1 & -\gamma_x & \eta_y \\ -\gamma_y & \gamma_x & 1 & \eta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (20) $$

where η_x, η_y and η_z are the position errors, γ_x, γ_y and γ_z are the angular errors. Using the MathLab program, substituting Eqns.(16),(17) and (18) into Eqn.(19), we can obtain the position errors of the FXYZ configuration as the followings.

$$ \begin{aligned} \eta_x &= [\delta_x(x) - y*\varepsilon_z(x) + z*\varepsilon_y(x)] + [\delta_x(y) + z*\varepsilon_y(y)] + [\delta_x(z)] \\ \eta_y &= [\delta_y(x) - z*\varepsilon_x(x)] + [\delta_y(y) - z*\varepsilon_x(y)] + [\delta_y(z)] \\ \eta_z &= [\delta_z(x) + y*\varepsilon_x(x)] + [\delta_z(y)] + [\delta_z(z)] \end{aligned} \qquad (21) $$

Eqn.(21) is the same as Eqn.(11).

Similarly, the error matrix ${}^{p}_{\tau}\boldsymbol{E}$ can be obtained for the configuration XFYZ, XYFZ and XYZF, and the position errors can be calculated by the MathLab program. The results are the same as Eqns. 12, 13, and 14 respectively.

6. Stacking model and Abbe offset

The displacement errors caused by the pitch, yaw and roll angular errors are the Abbe offset times the angular errors. The sign is determined by the right-hand-rule.

For the configuration FXYZ (Fig. 1), x-axis is mounted on a fixed base, y-axis is mounted on the x-axis and z-axis is mounted on the y-axis. Hence for x-axis movement, there is no Abbe offset on x and the angular error terms are $y*\varepsilon_x(x)$, $y*\varepsilon_z(x)$, $-z*\varepsilon_x(x)$ and $z*\varepsilon_y(x)$; for y-axis movement, there are no Abbe offset on x and y and the angular error terms are $-z*\varepsilon_x(y)$ and $z*\varepsilon_y(y)$; for z-axis movement, there are no Abbe offsets on x, y and z and there is no angular error term. This result is the same as Eqn. (11).

Similarly for the configuration XFYZ (Fig. 2), x-axis is mounted on a fixed base, y-axis is mounted on the x-axis and z-axis is mounted on the y-axis. Hence for x-axis movement, there are all 3 Abbe offsets and the angular error terms are $-x*\varepsilon_y(x)$, $x*\varepsilon_z(x)$, $y*\varepsilon_x(x)$, $-y*\varepsilon_z(x)$, $-z*\varepsilon_x(x)$ and $z*\varepsilon_y(x)$; for y-axis movement, there are no Abbe offsets on x and y and the angular error terms are $-z*\varepsilon_x(y)$ and $z*\varepsilon_y(y)$; for z-axis movement, there are no Abbe offsets on x, y and z and there is no angular error term. This result is the same as Eqn. (12).

Similarly for the configuration XYFZ (Fig. 3), x-axis is mounted on a fixed base, y-axis is mounted on the x-axis and z-axis is mounted on a fixed base. Hence for x-axis movement, there is no Abbe offset on y and the angular error terms are $-x*\varepsilon_y(x)$, $x*\varepsilon_z(x)$, $-z*\varepsilon_x(x)$ and $z*\varepsilon_y(x)$; for y-axis movement, there are all 3 Abbe offsets, and the angular terms are $x*\varepsilon_y(y)$, $x*\varepsilon_z(y)$, $y*\varepsilon_x(y)$, $-y*\varepsilon_z(y)$, $-z*\varepsilon_x(y)$ and $z*\varepsilon_y(y)$; for z-axis movement, there is no Abbe offset on x, y and z and no angular term. This result is the same as Eqn. (13).

Finally for the configuration XYZF (Fig. 4), x-axis is mounted on a fixed base, y-axis is mounted on the x-axis and z-axis is mounted on the y-axis and the spindle is fixed. Hence for x-axis movement, there are no Abbe offsets on x and y and the angular error terms are - $z*\varepsilon_x(x)$, and $z*\varepsilon_y(x)$; for y-axis movement, there is no Abbe offset on z, and the angular error terms are - $x*\varepsilon_y(y)$, $x*\varepsilon_z(y)$, $y*\varepsilon_x(y)$, - $y*\varepsilon_z(y)$; for z-axis movement, there are all 3 Abbe offsets and the angular error terms are - $x*\varepsilon_y(z)$, $x*\varepsilon_z(z)$, $y*\varepsilon_x(z)$, - $y*\varepsilon_z(z)$, - $z*\varepsilon_x(z)$ and $z*\varepsilon_y(z)$. The results are the same as Eqn. (14).

7. Formulae for the 4 body diagonal displacement errors

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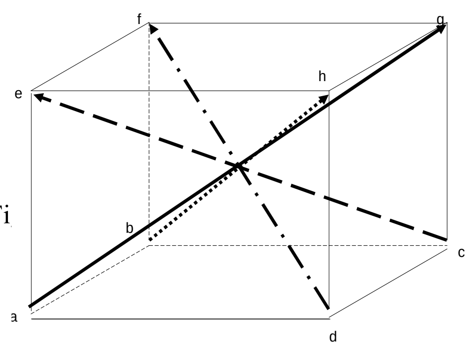

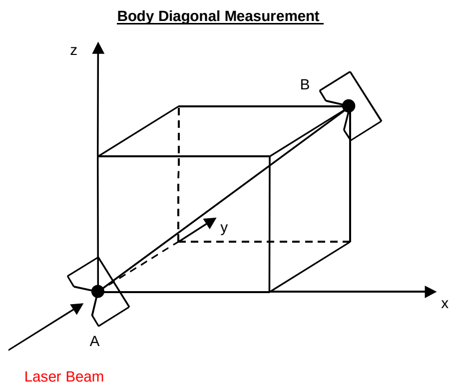

For the 4 body diagonal displacement measurement [1, 2], the measurement directions are ag. bh, ce, and df, as shown in Figure 5. The measurement is performed with the laser pointing along the body diagonal direction and the retroreflector moving along the body diagonal with a fixed increment as shown in Figure 6. For the configuration FXYZ, using Eqn. (11), the measured error DR at each increment can be expressed as:

$$\begin{aligned}\text{DRppp} = {} & a/r *\delta_x(x) + b/r *\delta_y(x) + c/r *\delta_z(x) \\ & + a/r*[\delta_x(y) + y\, S_{xy}] + b/r*\delta_y(y) + c/r*\delta_z(y) \\ & + a/r*[\delta_x(z) + z\, S_{zx}] + b/r*[\delta_y(z) + z\, S_{yz}] + c/r *\delta_z(z) \\ & + \varepsilon_y(x) *ac/r - \varepsilon_z(x) *ab/r + \varepsilon_y(y) *ac/r - \varepsilon_x(y) *bc/r.\end{aligned} \quad (22)$$

$$\begin{aligned}\text{DRnpp} = {} & -a/r *\delta_x(x) + b/r *\delta_y(x) + c/r *\delta_z(x) + \\ & - [\delta_x(y) + y\, S_{xy}] + b/r*\delta_y(y) + c/r*\delta_z(y) \\ & - a/r*[\delta_x(z) + z\, S_{zx}] + b/r*[\delta_y(z) + z\, S_{yz}] + c/r *\delta_z(z) \\ & - \varepsilon_y(x) *ac/r + \varepsilon_z(x) *ab/r - \varepsilon_y(y) *ac/r - \varepsilon_x(y) *bc/r.\end{aligned} \quad (23)$$

$$\begin{aligned}\text{DRpnp} = {} & a/r *\delta_x(x) - b/r *\delta_y(x) + c/r *\delta_z(x) \\ & + a/r*[\delta_x(y) + y\, S_{xy}] - b/r*\delta_y(y) + c/r*\delta_z(y) \\ & + a/r*[\delta_x(z) + z\, S_{zx}] - b/r*[\delta_y(z) + z\, S_{yz}] + c/r *\delta_z(z) \\ & + \varepsilon_y(x) *ac/r + \varepsilon_z(x) *ab/r + \varepsilon_y(y) *ac/r + \varepsilon_x(y) *bc/r.\end{aligned} \quad (24)$$

$$
\begin{aligned}
DRppn = {} & a/r * \delta_x(x) + b/r * \delta_y(x) - c/r * \delta_z(x) \\
& + a/r*[\delta_x(y) + y\, S_{xy}] + b/r*\delta_y(y) - c/r*\delta_z(y) \\
& + a/r*[\delta_x(z) + z\, S_{zx}] + b/r*[\delta_y(z) - z\, S_{yz}] + c/r * \delta_z(z) \\
& - \varepsilon_y(x) *ac/r - \varepsilon_z(x) *ab/r - \varepsilon_y(y) *ac/r + \varepsilon_x(y) *bc/r.
\end{aligned}
\tag{25}
$$

where the subscript ppp means body diagonal with all x, y and z positive; npp means body diagonal with x negative, y and z positive; pnp means body diagonal with y negative, x and z positive; and ppn means body diagonal with z negative, x and y positive.

Also a, b, c and r are increments in x, y, z and body diagonal directions respectively. The increment in body diagonal direction can be expressed as $r^2 = a^2 + b^2 + c^2$. Similar equations have also been derived in [7].

The 4 body diagonal displacement errors shown in Eqns. 22 to 25 are sensitive to all of the 9 linear errors and some angular errors. Hence it is a good measurement of the 3D volumetric positioning errors. The errors in the above equations may be positive or negative and they may cancel each other. However, the errors are statistical in nature, the probability that all of the errors will be cancelled in all of the positions and in all of the 4 body diagonals are theoretically possible but very unlikely. Hence it is indeed a quick measurement of volumetric positioning accuracy.

In the FXYZ configuration, as shown in Eqns. 22 to 25 there are 4 angular error terms, $\varepsilon_y(x)$*ac/r, $-\varepsilon_z(x)$*ab/r, $\varepsilon_y(y)$*ac/r and $-\varepsilon_x(y)$*bc/r. In the XFYZ configuration (Eqn. 12), most of the angular error terms are cancelled and only 2 angular error terms, $\varepsilon_y(y)$*ac/r and $-\varepsilon_x(y)$*bc/r are left. Similarly in the XYFZ configuration (Eqn. 13), only 2 angular error terms, $\varepsilon_z(x)$*ab/r and $-\varepsilon_x(x)$*bc/r are left. Finally in the XYZF configuration (Eqn. 14), there are 4 angular error terms, $\varepsilon_y(x)$*ac/r, $-\varepsilon_z(x)$*ab/r, $\varepsilon_y(y)$*ac/r and $-\varepsilon_x(y)$*bc/r exactly the same as in the FXYZ configuration. Since the configuration for most common horizontal machining centers and vertical machining centers are XFYZ and XYFZ respectively, we can conclude that the body diagonal displacement measurement are not sensitive to angular errors.

It is noted that if the 4 body diagonal displacement errors are small, then the machine errors are most likely very small. If the 4 body diagonal displacement errors are large, then the machine errors are large. However, because there are only 4 sets of data and 9 sets of errors, we do not have enough information to determine which errors are large. In order to determine where the large errors are, the sequential step diagonal measurement or laser vector technique [7] has been developed by Optodyne to collect 12 sets of data with the same 4 diagonal setups. Based on these data, all 3 displacement errors, 6 straightness errors and 3 squareness errors can be determined. Furthermore, the measured positioning errors can also be used to generate a 3D volumetric compensation table to correct the positioning errors and achieve higher positioning accuracy. Hence 3D volumetric positioning errors can be measured without incurring high costs and long machine tool down time.

8. Summary and conclusion

In summary, the 4 body diagonal displacement measurement is a quick measure of all the 9 linear errors, 3 displacement errors, 6 straightness errors, 3 squareness errors and some of the angular errors. The sequential step diagonal measurement collects 12 sets of data to solve for all the 9 linear errors and 3 squareness errors. With additional 3 sets of linear displacement measurement for each axis, all the angular errors (except roll) can be determined [7].

In conclusion, for the XFYZ and XYFZ configurations, there are only 2 angular error terms. Hence the body diagonal displacement measurement is not sensitive to angular errors. It is a good quick check of the volumetric positioning accuracy including 3 displacement errors, 6 straightness errors and 3 squareness errors. The sequential body diagonal or vector technique can be used to determine and compensate the volumetric positioning errors.

References

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