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## ISSUES IN LASER STEP DIAGONAL MEASUREMENT AND THEIR REMEDIES

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### ABSTRACT

The laser step diagonal measurement modifies the diagonal displacement measurement by executing a diagonal as a sequence of single-axis motions. It has been claimed that it enables the identification of all the volumetric error components including linear errors, straightness and squareness errors. In this paper, we show that the conventional formulation of the step diagonal measurement is valid only when implicit assumptions related to the configuration of laser and mirror setups are met, and that its inherent problem is that it is generally not possible to meet these conditions by the adjustment of the setup. To address this problem, we propose a new formulation of the step diagonal measurement, in order to accurately identify volumetric errors even under the existence of setup errors. The effectiveness of the proposed modified identification scheme is investigated experimentally by an application example to a high-precision machine tool.

**Key Words:** Step-diagonal measurement, volumetric errors, machine tool, laser interferometer.

### INTRODUCTION

To meet increasing demands in the manufacturing of optical parts or electronic parts, high-precision and ultra-precision machine tools have been rapidly introduced into the market in recent years. To ensure the motion accuracy over the entire three-dimensional workspace of such a machine tool, it is important to evaluate all the volumetric errors including 3 linear displacement errors, 6 straightness errors and 3 squareness errors (Wang, 2005). For the measurement of linear displacement errors, laser interferometers of the resolution sufficient to measure high-precision and ultra-precision machines are now available. It is, however, relatively difficult and time-consuming to measure other volumetric errors such as straightness and squareness errors. Typically, straightness and squareness errors are measured by using a high-precision displacement sensor and an artifact such as a straight edge or a square edge. For the measurement on high-precision or ultra-precision machines, the artifact whose geometric and dimensional accuracies of the artifact are guaranteed to be higher than the accuracy of the measured machine is needed, which requires higher measurement cost. Furthermore, since the measurement is one-

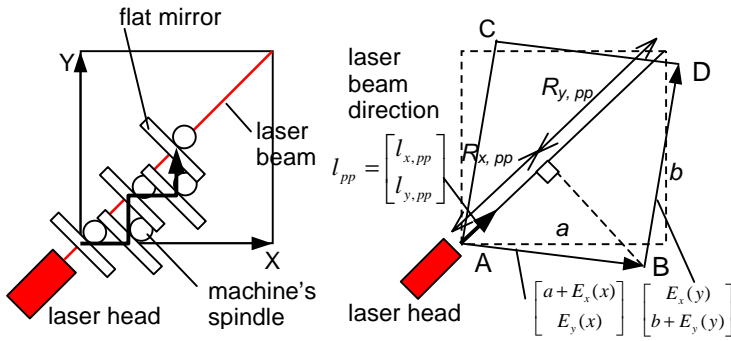
dimensional and its path is restricted to a line or a square, an operator must change the setup of a sensor and an artifact every time for the measurement of each different error component. Dual-beam laser systems or autocollimators to measure straightness and squareness errors do not require an artifact such as a straight edge, but it is the same in that a different setup is needed to measure each different error component.

For quicker, lower-cost evaluation of volumetric errors of a machine tool, the step diagonal measurement has been proposed by Wang (2000). In the diagonal measurement described in B5.54 and ISO230-6, the machine moves its X, Y, and Z axes simultaneously along each body diagonal. In the step diagonal measurement, each axis is moved one at a time along the “zig-zag” path toward the body diagonal direction. Wang claimed that additional data enables the identification of all the volumetric errors from step diagonal measurements.

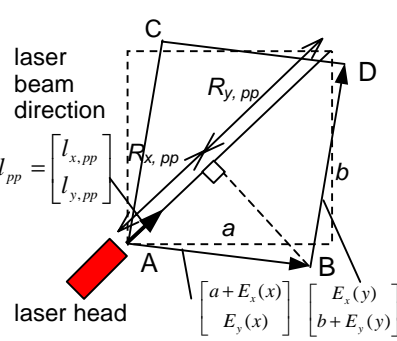
The objective of this paper is to discuss the validity of the error identification based on step diagonal measurements. In this paper, we show that the formulation of the step diagonal measurement presented by Wang (2000) is valid only when implicit assumptions related to laser and mirror setups are met, and that its inherent problem is that it is generally not possible to meet these conditions by the adjustment of the setup. As for issues in the laser step diagonal measurement, Chapman (2003) discussed the misalignment of the mirror, and Soons (2005) formulated the effect of angular errors on identification accuracies. This paper will present rather critical issues with Wang’s formulation of the step diagonal measurement. Then, as remedies for these issues, we will propose a new formulation of the step diagonal measurement, in order to accurately identify each volumetric error even under the existence of setup errors. The validity of the discussion on issues in step diagonal measurements and the effectiveness of the proposed modified identification scheme will be investigated experimentally by showing application examples to a high-precision machine tool.

### CONVENTIONAL FORMULATION OF STEP DIAGONAL MEASUREMENT

First, this section briefly reviews the conventional formulation of the identification of volumetric errors based on the step diagonal measurement presented by Wang (2000). To simplify



**Figure 1. The schematics of two-dimensional step-diagonal measurement**



**Figure 2. Volumetric errors and diagonal displacements**

the discussion, this paper only considers the two-dimensional step-diagonal measurement in the XY plane. The extension to a three-dimensional case is straightforward.

Figure 1 illustrates the setup of 2D step-diagonal measurement. As the machine spindle, where a plane mirror is attached, moves along a “zig-zag” path, the moving distance along the face diagonal is measured by using a laser interferometer. As illustrated in Fig. 2, suppose that the laser is aligned to the direction represented by the unit vector  $l_{pp} = [l_{x,pp}, l_{y,pp}]$  (this setup is referred to as *pp* measurement hereafter). Define  $E_x(x)$  and  $E_y(x)$  as the positioning error in *x*- and *y*-directions, respectively, due to the motion toward *x* direction (i.e.  $A \rightarrow B$ ).  $E_x(y)$  and  $E_y(y)$  are defined similarly. In this paper, these four error components are called volumetric errors. The distance measured by a laser interferometer with the motion toward *x* ( $A \rightarrow B$ ) and *y* ( $B \rightarrow D$ ) are given by  $R_{x,pp}$  and  $R_{y,pp}$ , respectively. A similar measurement is done as the laser is aligned along the diagonal  $BC$  (this setup is referred to as *np* measurement).  $R_{x,np}$  and  $R_{y,np}$  are defined similarly. Then, we have (Wang, 2000):

$$\begin{bmatrix} l_{x,pp} & l_{y,pp} & 0 & 0 \\ 0 & 0 & l_{x,np} & l_{y,np} \\ -l_{x,np} & -l_{y,np} & 0 & 0 \\ 0 & 0 & l_{x,pp} & l_{y,pp} \end{bmatrix} \begin{bmatrix} a + E_x(x) \\ E_y(x) \\ E_x(y) \\ b + E_y(y) \end{bmatrix} = \begin{bmatrix} R_{x,pp} \\ R_{y,pp} \\ R_{x,np} \\ R_{y,np} \end{bmatrix} \quad (1)$$

Assume nominal laser directions, i.e.

$$[l_{x,pp} \ l_{y,pp}] = [a \ b] / \sqrt{a^2 + b^2}, \quad [l_{x,np} \ l_{y,np}] = [-a \ b] / \sqrt{a^2 + b^2} \quad (2)$$

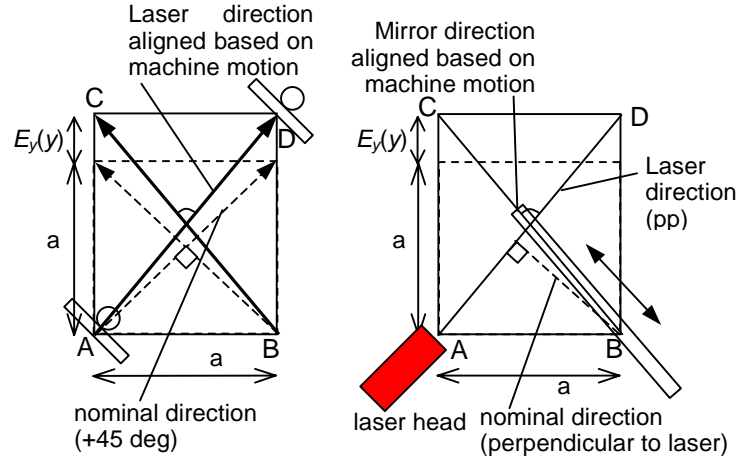
Then, volumetric errors,  $E_x(x)$ ,  $E_y(x)$ ,  $E_x(y)$  and  $E_y(y)$ , can be estimated from  $R_{x,pp}$ ,  $R_{y,pp}$ ,  $R_{x,np}$  and  $R_{y,np}$  by solving Eq. (1).

## ISSUES IN VOLUMETRIC ERROR IDENTIFICATION BASED ON STEP DIAGONAL MEASUREMENT

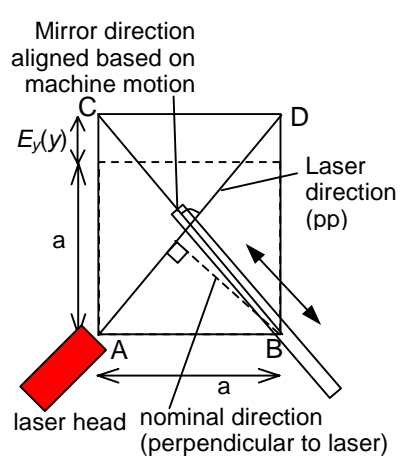
An inherent issue in the identification of volumetric errors based on Eq. (1) is that Eq.(1) is valid only when the following conditions are satisfied, and if they are not satisfied, they likely result in non-negligible identification errors.

- (1) Laser beam directions must be precisely aligned to nominal directions, Eq. (2).
- (2) The flat mirror must be precisely aligned perpendicular to the laser beam direction.
- (3) The angular errors of the machine are negligibly small.

In practice, no matter how careful an operator sets up the laser head and the mirror, it is simply *not possible* to satisfy the conditions (1) and (2), when the positioning accuracy of the machine is unknown. We claim that this is an inherent critical issue with the conventional formulation.



**Figure 3. Misalignment of laser beam direction caused by machine's volumetric error**



**Figure 4. Misalignment of mirror direction caused by machine's volumetric error**

**Table 1. Simulated volumetric errors identification by conventional formulation of step-diagonal measurements**

	Case (a)		Case (b)		Case (c)	
	given	estimated	given	estimated	given	estimated
$E_x(x)$	0	-0.05	0.1	0.1	0	-0.0002
$E_y(x)$	0	0	0	0	0	0
$E_x(y)$	0	0	0	0	0.1	0.1000
$E_y(y)$	0.1	0.15	0.1	0.1	0	0.0002

The effect of angular errors on the identification accuracy was discussed in details by Soons (2005), and thus it is not considered in this paper.

## Misalignment of laser beam directions

Except for a special case, the laser beam direction in *pp* and *np* measurements can be only aligned *based on the motion of the machine to be measured*. That is, in a typical setup, the laser beam direction is aligned such that it becomes parallel to the machine's diagonal. For example, when the machine moves from  $A$  to  $D$  in Fig. 2, the laser direction is adjusted such that the deviation of the laser spot location on the mirror is minimized (the alignment can be done more precisely if a quad-detector is used). Here, if the machine has volumetric errors and they are unknown, it is not possible to align the laser beam perfectly to the nominal direction. An illustrative example is shown in Fig. 3. This example assumes that  $a=b$  and  $E_x(y) > 0$ ,  $E_x(x) = E_y(x) = E_y(y) = 0$ . Since the laser beam is aligned to the machine's diagonal, laser directions in *pp*- and *np*- measurements do not cross perpendicularly. Since the identification based on Eq. (2) assumes the perpendicularity of laser beam directions (in case of  $a=b$ ), it potentially causes an identification error.

Notice that a small misalignment error of laser direction does not affect much measured displacements,  $R_{x,pp}$ ,  $R_{y,pp}$ ,  $R_{x,np}$  and  $R_{y,np}$ , as has been also mentioned by Wang (2000). However, an error in  $l_{x,np}$ ,  $l_{y,np}$ ,  $l_{x,pp}$ ,  $l_{y,pp}$  may impose non-negligible identification errors when solving Eq.(1).

## Illustrative simulation examples

To show that the misalignment error of laser may potentially results in a large identification error, illustrative numerical simulations are presented. Table 1 compares given and estimated volumetric errors. In numerical simulations, laser direc-

tions are assumed to be aligned perfectly to the machine's diagonal directions. The mirror is assumed to be perfectly aligned perpendicular to the laser beam direction. The estimates are computed by solving Eq. (1) with the nominal values of  $l_{x,np}$ ,  $l_{y,np}$ ,  $l_{x,pp}$ ,  $l_{y,pp}$ . The estimates are "rotated" such that the estimated  $E_y(x)$  becomes zero to avoid the redundancy.

In the case (a), the estimates contain a large estimation error, while the cases (b) and (c) have almost no error. This can be understood by observing that laser directions (i.e. diagonal directions) are not perpendicular to each other in case (a), while they "happen to be" approximately perpendicular in the cases (b) and (c). In case (a), estimation errors are as large as half of the given error, which are clearly not negligibly small.

### Misalignment of mirror directions

Similarly, the direction of the flat mirror can be only aligned based on the motion of the machine to be measured, as illustrated in Fig. 4. For example, in the pp measurement, the machine is moved from B to C, and then the mirror direction is adjusted such that the measured diagonal distance becomes approximately equal at both ends of the mirror. This adjustment aligns the mirror ideally parallel to the diagonal direction, but it does not ensure the perpendicularity of laser and mirror directions. Since Eq.(1) assumes the perpendicularity of laser and mirror directions, it potentially results in estimation errors. Also, notice that if the mirror is perfectly aligned to the diagonal direction by the adjustment above, it simply makes  $R_{x,pp}=R_{y,pp}$  and  $R_{x,np}=R_{y,np}$ . It means that the step diagonal measurement only provides diagonal distances, and thus the error identification based on Eq. (1) will obviously fail.

### A NEW FORMULATION OF STEP DIAGONAL MEASUREMENT

It is not possible to identify all the volumetric errors from  $R_{x,pp}$ ,  $R_{y,pp}$ ,  $R_{x,np}$  and  $R_{y,np}$  when laser directions and mirror directions are not known. As a remedy, we propose a new formulation of the step diagonal measurement to identify  $E_y(x)$  and  $E_x(y)$ , under the assumption that linear error components,  $E_x(x)$  and  $E_y(y)$ , are known by direct measurement.

(1) Identification of  $E_x(y)$  (under the assumption of  $E_y(x)=0$ )

From the geometric relationship in pp measurement, when it is assumed that  $E_y(x)=0$ ,  $E_x(y)$  can be estimated as follows:

$$\hat{E}_x^0(y) = \sqrt{R_{pp}^2 - (b + E_y(y))^2} - (a + E_x(x)) \quad (3)$$

where  $R_{pp}=R_{x,pp}+R_{y,pp}$  represents the measured diagonal distance in pp measurement. Unlike Eq. (1), Eq. (3) is not explicitly dependent on laser beam directions,  $l_{pp}$  and  $l_{np}$ , and thus the sensitivity to the misalignment error of laser directions is far smaller than the conventional formulation. Also note that since Eq. (3) only uses diagonal distances, they are insensitive to alignment errors of mirror direction.

(2) Identification of  $E_y(x)$

In Fig. 6,  $\alpha$  denotes the angle between the mirror and the laser beam in the pp measurement, which is unknown.  $\delta$  denotes the distance  $\delta=0.5R_{pp}-R_{x,pp}$ . In the first block of step diagonal measurement where it can be assumed that  $E_y(x)=0$  (notice that since  $E_y(x)$  can be defined only relatively, it is possible to assume this in one block), we have:

$$\sin \alpha = -\frac{a + E_x(x)}{0.5R_{pp} - \delta(1)} \cos \alpha_2 \quad (4)$$

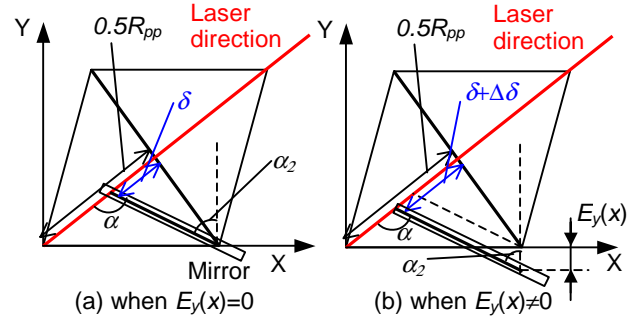


Figure 6. Identification of  $E_y(x)$

where  $\alpha_2$  denotes the angle of the mirror direction with respect to the vertical direction. Without loss of generality, we can assume that  $\alpha_2=\pi/4$ . Under the assumption that there is no angular error in the machine's motion, in the  $i$ -th block,  $E_y(x)$  can be estimated as follows:

$$\hat{E}_y(x)(i) = -\frac{\sin a}{\sin a_2} \cdot \Delta\delta(i) \quad (5)$$

where  $\Delta\delta(i)$  is the displacement in the measured  $\delta=0.5R_{pp}(i)-R_{x,pp}(i)$  from its value in the first block,  $\delta(1)$ .

Notice that this formulation is also insensitive to the laser direction. Furthermore, under the assumption that the misalignment of the mirror direction is constant in all the blocks, the misalignment of the mirror direction does not affect the estimates of  $E_y(x)$ .

(3) Modification of  $E_x(y)$

Based on the estimated  $E_y(x)$ ,  $E_x^0(y)$  estimated in Step (1) is modified by

$$\hat{E}_x(y) = (b + E_y(y)) \cos \delta + \hat{E}_x^0(y) \sin \delta \quad (6)$$

where  $\delta = \tan^{-1}(\hat{E}_y(x)/(a + E_x(x)))$

### CASE STUDY

The effectiveness of the proposed modified identification scheme is investigated experimentally by an application example to a three-axis vertical-type high-precision milling machine. Its axes are all driven by a linear motor with a hydrostatic guideway. Its positioning resolution is 10 nm in all the axes. The stroke is X100mm×Y100mm. For the laser measurement, a laser doppler displacement meter, MCV-500 by Optodyne, Inc. is used. Laser beam directions are aligned by using a quad-detector, LD42 by Optodyne, Inc. The step diagonal measurements are done with the step size  $a=b=10$  mm, over the entire range of 60 mm×60 mm (i.e. 6 blocks in X and Y directions).

First, volumetric errors are estimated by the conventional formulation using step-diagonal measurements only. Figure 7(a)(b) show estimated linear positioning errors in X and Y directions,  $P_x(x)$  and  $P_y(y)$ , with respect to each reference point.  $P_x(x)$  and  $P_y(y)$  are given by the accumulation of  $E_x(x)$  and  $E_y(y)$ . Their measured values, obtained by using the same laser interferometer aligned directly toward X- and Y-directions, are also shown. In each measurement, the same measurement is repeated by three times. Figure 7 plots the mean of estimated and measured errors, as well as their variation at each measurement point by horizontal parallel lines. The mean positioning error measured by the laser interferometer is +1.14  $\mu\text{m}$  over 60 mm in the X direction, and +0.69  $\mu\text{m}$  over 60 mm in the Y

direction. The conventional identification scheme results in a large estimation error in both X and Y directions. The estimate of mean positioning error is  $-0.74 \mu\text{m}$  over 60 mm in the X direction, and  $+2.27 \mu\text{m}$  over 60 mm in the Y direction.

Then, errors in the normal direction,  $E_y(x)$  and  $E_x(y)$ , are estimated based on the proposed formulation, by using measured profiles in pp-, np-, X-, and Y-directions. Estimated and measured mean profiles of the accumulated positioning error in the normal direction,  $P_y(x)$  and  $P_x(y)$ , are compared in Fig. 8(a)(b). Here, measured profiles are obtained by using a cross grid encoder (KGM), KGM182 by Heidenhain. In Fig. 8, the mean profile of the estimates obtained from pp- and np-measurements by using the conventional formulation are also plotted. Table 2 summarizes measured and estimated straightness and squareness errors. Figure 8 and Table 2 show a good match between measured and estimated volumetric errors obtained by step diagonal measurements. The mean of estimation error, i.e. the mean of difference between measured and estimated errors obtained based on the proposed formulation, is  $<0.01 \mu\text{m}$  for  $P_y(x)$ , and  $0.07 \mu\text{m}$  for  $P_x(y)$ . The maximum estimation error is  $0.07 \mu\text{m}$  for  $P_y(x)$ , and  $0.18 \mu\text{m}$  for  $P_x(y)$ .

It should be noted that the conventional formulation gives a fairly good estimation accuracy for  $P_y(x)$  or  $P_x(y)$ , similarly as the proposed formulation, although it results in a large estimation error for linear errors,  $P_x(x)$  or  $P_y(y)$ . This implies that the proposed formulation is essentially the simplification of the formulation (1), when the estimation of  $E_x(x)$  and  $E_y(y)$  is ignored.

## CONCLUSION

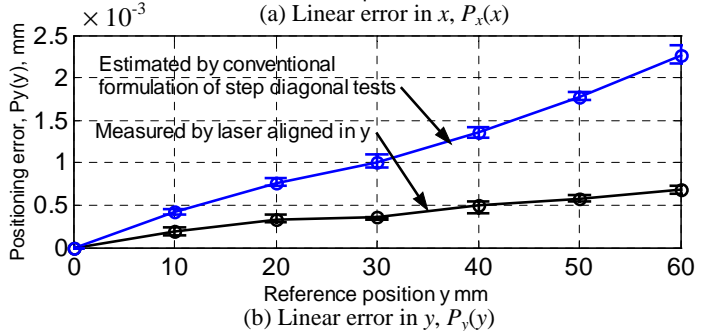
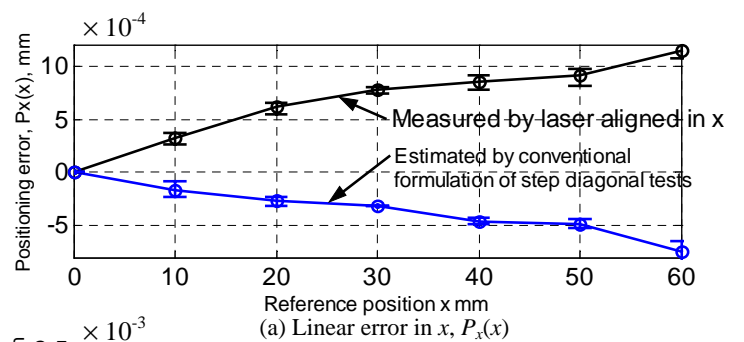
An inherent problem with the conventional formulation of the step diagonal measurement proposed by Wang (200) is that it is valid only when implicit assumptions related to the configuration of laser and mirror setups are met, and that it is generally not possible to meet these conditions by the adjustment of the setup. The new formulation proposed in this paper suggests that linear positioning errors must be independently measured, and then straightness (squareness) error components can be identified by using step-diagonal measurements even under the existence of setup errors. As an application example, the proposed scheme was applied to estimate two-dimensional volumetric errors on a high-precision milling machine of the positioning resolution of 10 nm. Experimental results indicated that the squareness error of X and Y axes ( $1.22 \mu\text{m}$  over 60mm measured by the KGM) was estimated with an estimation error of about  $0.1 \mu\text{m}$ .

## ACKNOWLEDGMENT

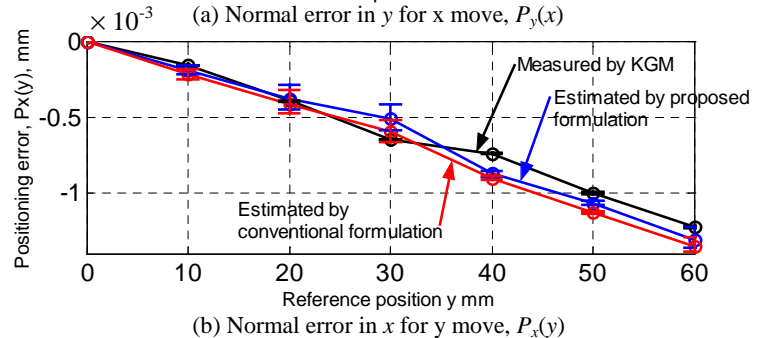
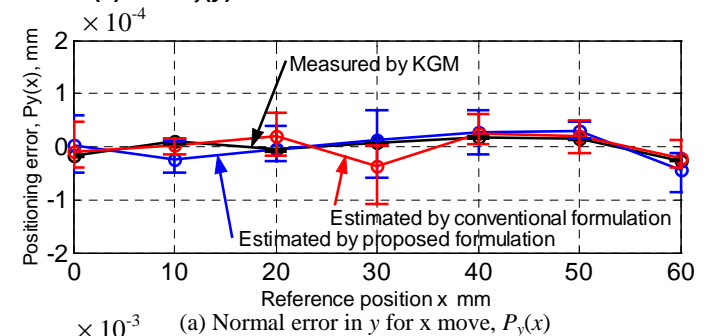
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**Figure 7. Measured and estimated volumetric errors,  $P_x(x)$  and  $P_y(y)$**



**Figure 8. Measured and estimated volumetric errors,  $P_y(x)$  and  $P_x(y)$**

**Table 2. Measured and estimated straightness errors (in X and Y) and squareness errors**

	Measured by KGM	Estimated by proposed formulation	Estimated by conventional formulation
Straightness error in X	$0.05 \mu\text{m}$	$0.07 \mu\text{m}$	$0.06 \mu\text{m}$
Straightness error in Y	$0.11 \mu\text{m}$	$0.15 \mu\text{m}$	$0.08 \mu\text{m}$
Squareness error in XY	$-1.22 \mu\text{m}$	$-1.32 \mu\text{m}$	$-1.37 \mu\text{m}$

\* All the errors are over the range of 60 mm.